

Section 4.1 Absolute Extrema (Minimum Homework: 1, 3, 5, 7, 9, 11, 13, 17, 19, 23, 25)

We cover the concept of the absolute maximum and absolute minimum in section 4.1.

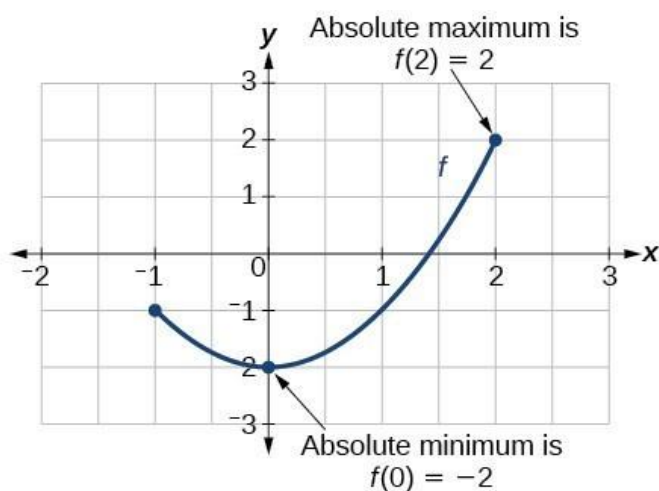
Given a graph of a continuous function over a closed interval: (We only consider graphs that are bound on each side when we find absolute maximum and absolute minimum values.)

- **Absolute maximum:** y-coordinate of the highest point
- **Absolute minimum:** y-coordinate of the lowest point.

The absolute maximum and absolute minimum points are marked on the graph below.

It would be proper to write the following:

- There is an absolute maximum of $y = 2$, which occurs when $x = 2$.
- There is an absolute minimum of $y = -2$, which occurs when $x = 0$.
(It would be correct to write $f(x)$ instead of y)



We need to be able to find the absolute maximum and absolute minimum of values of a function over a given interval $[a,b]$.

Here are the steps:

- 1) Find $f'(x)$
- 2) Solve $f'(x) = 0$
- 3) Create a table with two columns.

x Left column	$f(x)$ Right column
a	
b	
Any answer from step 2 in the interval $[a,b]$	
There will be as many extra rows as answers to part 2 in the given interval $[a,b]$	

4) Substitute the values in the left column into the ORIGINAL function to complete the right column.

x Left column	$f(x)$ Right column
a	$f(a)$
b	$f(b)$
Any answer from step 2 in the interval $[a,b]$	$f(\#)$

5) Write Answer

Absolute maximum – the largest value in the right column

Absolute minimum – smallest value in the right column.

Example: $f(x) = x^3 - 3x^2$; $[1,4]$

Find the absolute maximum and absolute minimum of the function under the given interval.

1) Find $f'(x)$

$$f'(x) = 3x^2 - 6x$$

2) Solve $f'(x) = 0$

$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$3x = 0 \quad x - 2 = 0$$

$$x = 0 \quad x = 2$$

3) Create a table with two columns.

Only include

1,2 and 4 in the left column of the table. Do not include 0 as it is not between

x	$f(x)$
1	
2	
4	

4)

x	$f(x)$
1	$f(1) = (1)^3 - 3(1)^2 = -2$
2	$f(2) = (2)^3 - 3(2)^2 = -4$
4	$f(4) = (4)^3 - 3(4)^2 = 16$

5) Write Answer

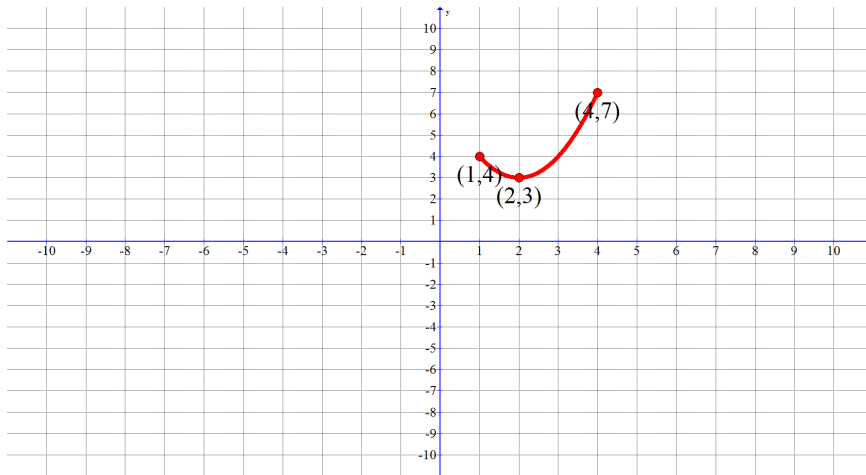
absolute maximum of $y = 16$, which occurs when $x = 4$.

absolute minimum of $y = -4$, which occurs when $x = 2$.

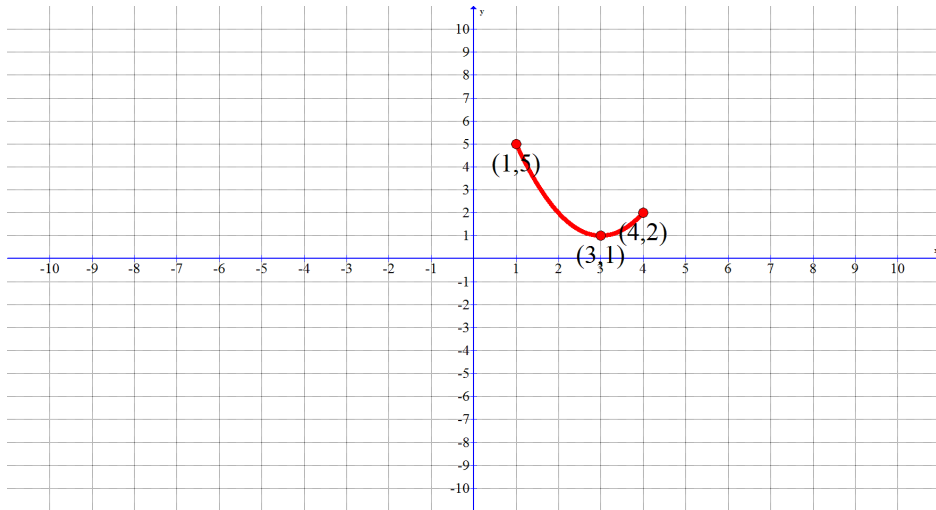
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#1-9: Find the absolute maximum and absolute minimum

1)



2)

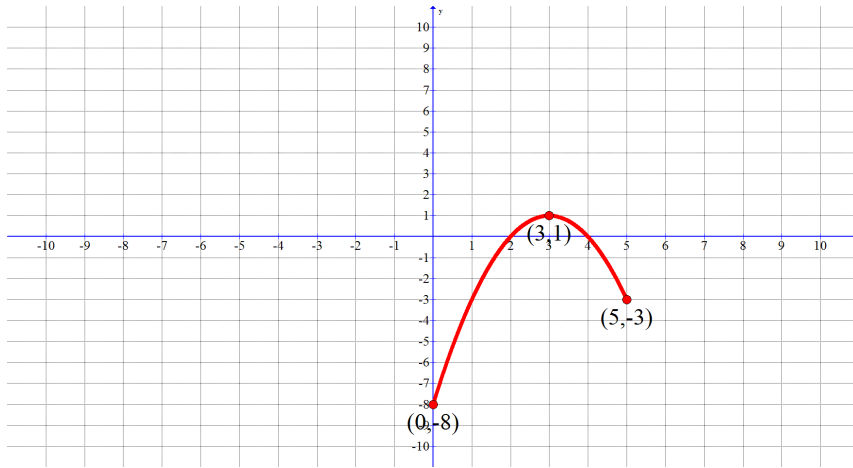


Absolute maximum of $y = 5$, which occurs when $x = 1$.

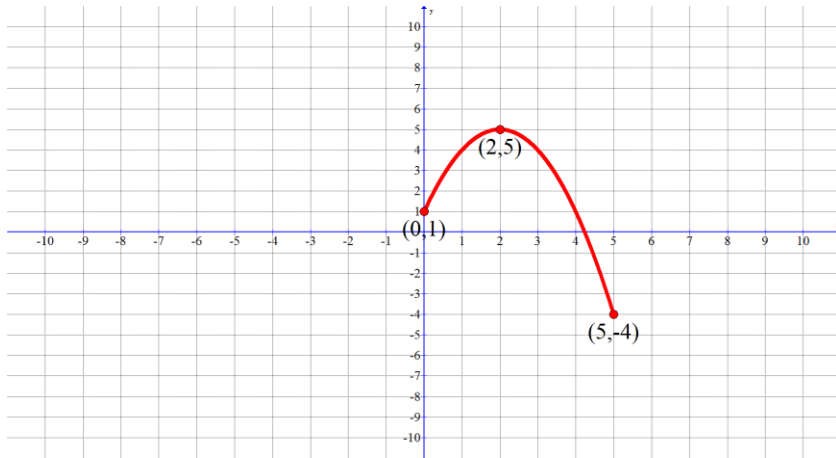
Absolute minimum of $y = 1$, which occurs when $x = 3$.

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3)



4)

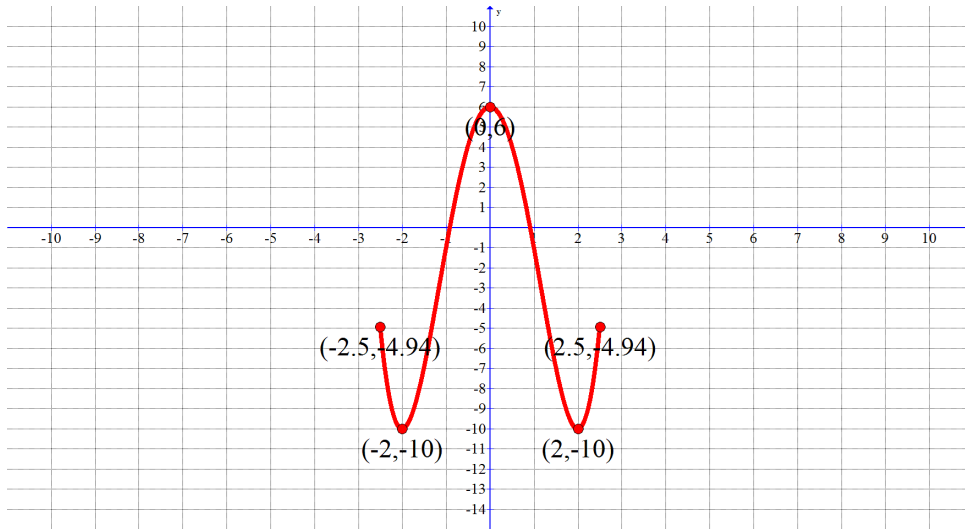


Absolute maximum of $y = 5$, which occurs when $x = 2$.

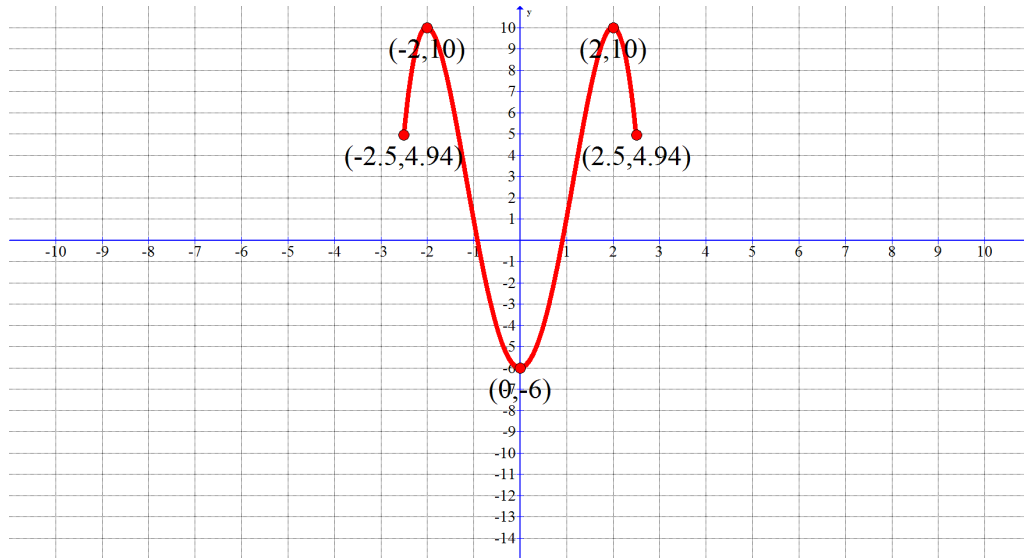
Absolute minimum of $y = -4$, which occurs when $x = 5$.

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5)



6)

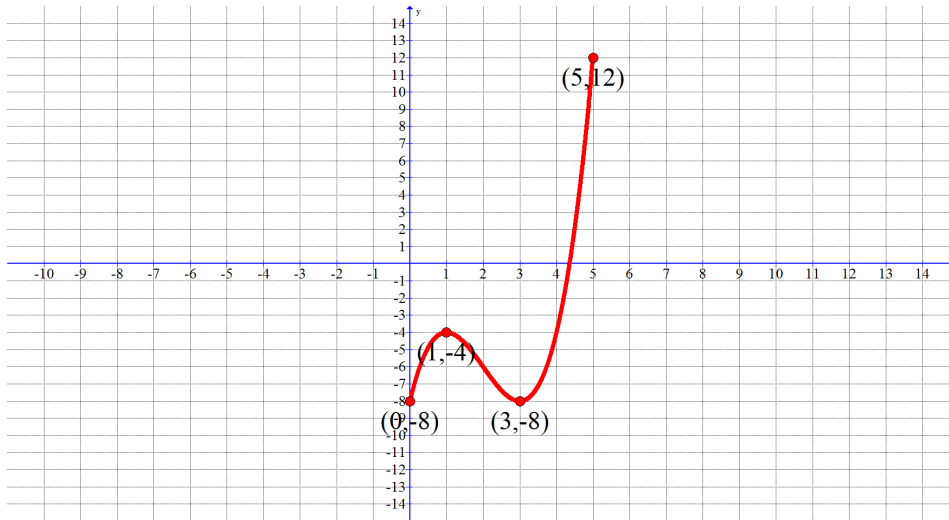


Absolute maximum of $y = 10$, which occurs when $x = -2, 2$.

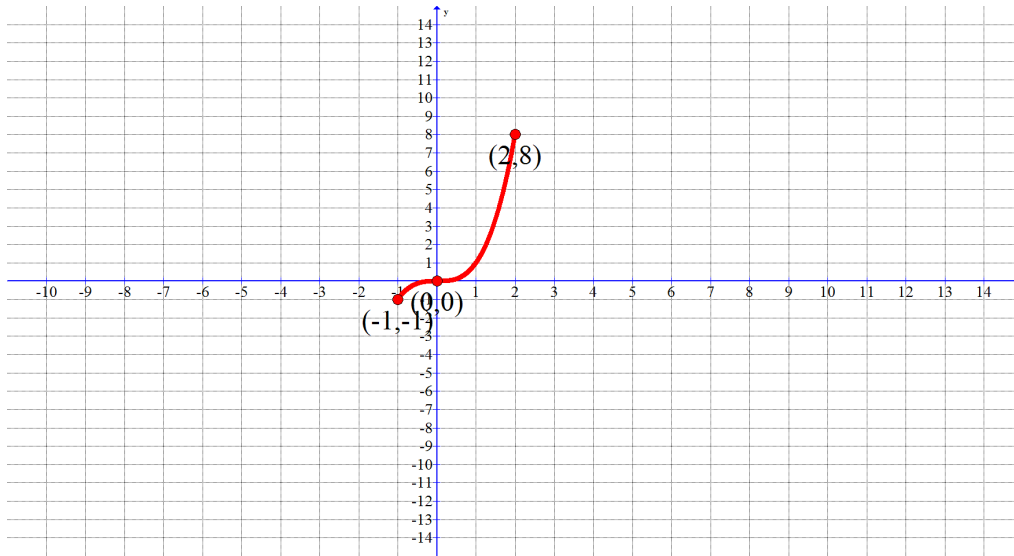
Absolute minimum of $y = -6$, which occurs when $x = 0$.

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7)



8)

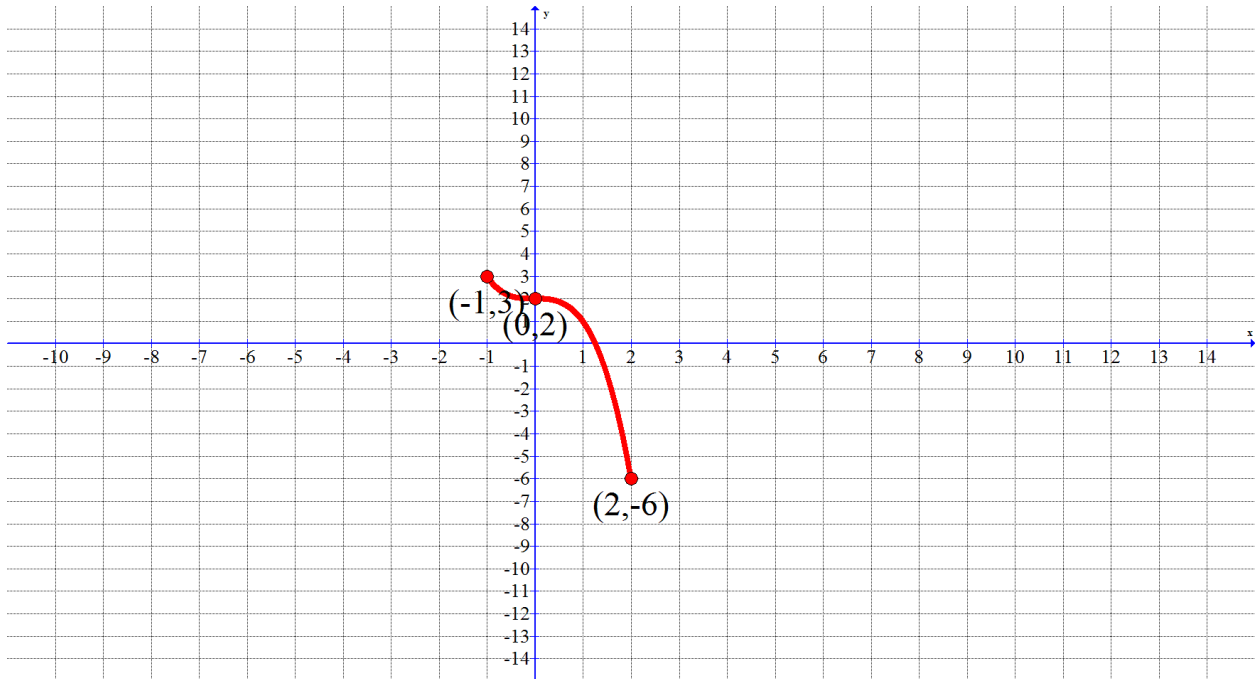


Absolute maximum of $y = 8$, which occurs when $x = 2$.

Absolute minimum of $y = -1$, which occurs when $x = -1$.

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9)



(Minimum Homework: 1, 3, 5, 7, 9, 11, 13, 17, 19, 23, 25)

#10-27: Find the absolute maximum and absolute minimum of the function under the given interval.

11) $f(x) = x^2 - 6x + 4$; $[-5,5]$

10) $f(x) = x^2 - 2x + 5$; $[-3,3]$

1) Find $f'(x)$

2) Solve $f'(x) = 0$

3) Create a table with two columns.

x	$f(x)$

4)

x	$f(x)$

5) Write Answer

Absolute maximum of $y = 20$, which occurs when $x = -3$.

Absolute minimum of $y = 4$, which occurs when $x = 1$.

13) $f(x) = x^3 + 6x^2; [-2,1]$

12) $f(x) = x^3 - 6x^2; [-2,2]$

1) Find $f'(x)$

2) Solve $f'(x) = 0$

3) Create a table with two columns.

x	$f(x)$

4)

x	$f(x)$

5) Write Answer

Absolute maximum of $y = 0$, which occurs when $x = 0$.

Absolute minimum of $y = -32$, which occurs when $x = -2$.

14) $f(x) = x^3 - 3x^2$; $[-1,3]$
 $3x^2 + 2$; $[-1,5]$

15) $f(x) = x^3 -$

17) $f(x) = 3x^4 - 4x^3$; $[-2,3]$

16) $f(x) = x^4 - x^3 + 5$; $[-2,2]$

1) Find $f'(x)$

2) Solve $f'(x) = 0$

3) Create a table with two columns.

x	$f(x)$

4)

x	$f(x)$

5) Write Answer

Absolute maximum of $y = 29$, which occurs when $x = -2$. Absolute minimum of $y = \frac{1253}{256}$, which occurs when $x = \frac{3}{4}$.

19) $f(x) = (x^2 - 16)^3; [-2,2]$

18) $f(x) = (x^2 - 9)^4; [0,2]$

1) Find $f'(x)$

2) Solve $f'(x) = 0$

3) Create a table with two columns.

x	$f(x)$

4)

x	$f(x)$

5) Write Answer

Absolute maximum of $y = 6561$, which occurs when $x = 0$.

Absolute minimum of $y = 625$, which occurs when $x = 2$.

20) $f(x) = \sqrt[3]{x}$; $[-1,2]$

21) $f(x) = \sqrt[5]{x}$; $[-3,2]$

23) $f(x) = 2xe^x$; $[0,3]$

22) $f(x) = xe^x$; $[-3,3]$

1) Find $f'(x)$

2) Solve $f'(x) = 0$

3) Create a table with two columns.

x	$f(x)$

4)

x	$f(x)$

5) Write Answer

Absolute maximum of $y = 3e^3$, which occurs when $x = 3$.

Absolute minimum of $y = -1/e$, which occurs when $x = -1$.

25) $f(x) = e^{x^2}; [-2,1]$

24) $f(x) = e^{3x^2}; [-1,1]$

1) Find $f'(x)$

2) Solve $f'(x) = 0$

3) Create a table with two columns.

x	$f(x)$

4)

x	$f(x)$

5) Write Answer

Absolute maximum of $y = e^3$, which occurs when $x = -1, 1$.

Absolute minimum of $y = 1$, which occurs when $x = 0$.

26) $f(x) = x^3 e^x; [-3,1]$

27) $f(x) = x^2 e^x; [-3,1]$